DEPARTMENT OF COMMERCE
SINCLAIR WEEKS, Secretary

WEATHER BUREAU
F. W. REICHELDERFER, Chief

# MONTHLY WEATHER REVIEW

JAMES E. CASKEY, JR., Editor

Volume 86
Number 7

JULY 1958

Closed September 15, 1958
Issued October 15, 1958

## THE ERROR IN NUMERICAL FORECASTS DUE TO RETROGRESSION OF ULTRA-LONG WAVES

PAUL M. WOLFF, CDR, U. S. NAVY [1]

Joint Numerical Weather Prediction Unit, Suitland, Md.

[Manuscript Received April 25, 1958; revised June 3, 1958]

ABSTRACT

Error charts for the numerical barotropic forecasts prepared at the Joint Numerical Weather Prediction Unit since October 1957 have revealed retrogressive patterns of very long wavelength. These errors are shown to be due to changes in the large-scale components predicted by the numerical model. These components are actually quasi-stationary in the atmosphere. Forecasts prepared with an approximation to these components held unchanged show significantly increased accuracy. Finally, some of the difficulties in developing a more acceptable physical approach to this problem are outlined.

### 1. INTRODUCTION

Beginning in October 1957 500-mb. barotropic forecasts by the Joint Numerical Weather Prediction Unit were prepared on a hemispheric basis with boundaries in the Tropics. The smaller area of previous forecasts had boundaries in meteorologically active locations, and it was widely anticipated that this expansion would be accompanied by a large reduction in gross error. Instead, the new error patterns were of very long wavelength and showed westward motion during the forecast.

Figure 1 is a 48-hour error chart in which the type of error treated here is prominent. The scale of the pattern is much greater than that of the corresponding 48-hour observed height changes. The largest scale error is positive over the central Pacific Ocean and negative over the eastern Atlantic and western Europe. The persistence in location of these errors from day to day has been so high that the variations in shape and intensity due to other causes are not large enough to cause a reversal of sign in these areas. That is, while the magnitude

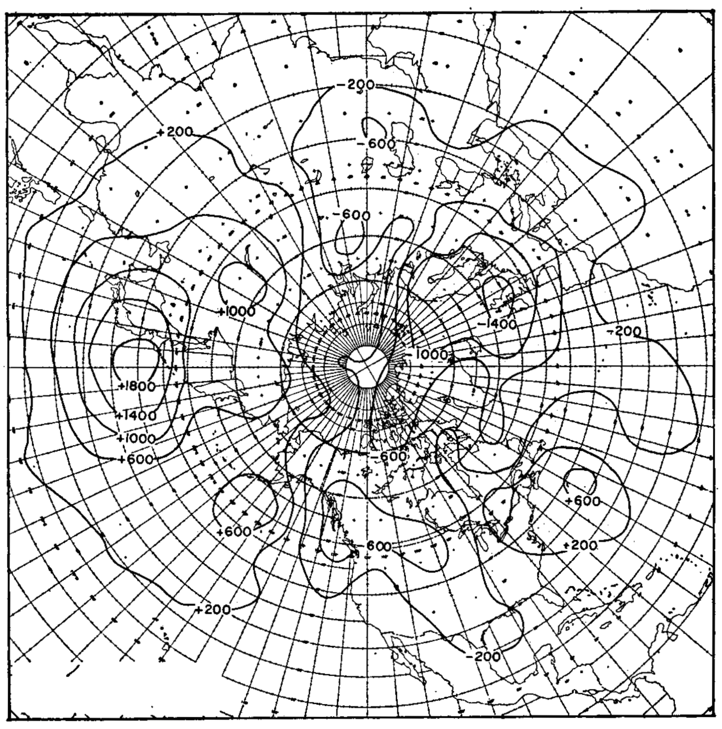


FIGURE 1.—48-hour error in feet. JNWP Unit operational forecast from 0000 GMT, January 16, 1958.

[1] Any opinions expressed by the author are his own and do not necessarily reflect the views of the Navy Department. Author's present affiliation: Navy Numerical Weather Problems Group, Fleet Weather Central, Suitland, Md.

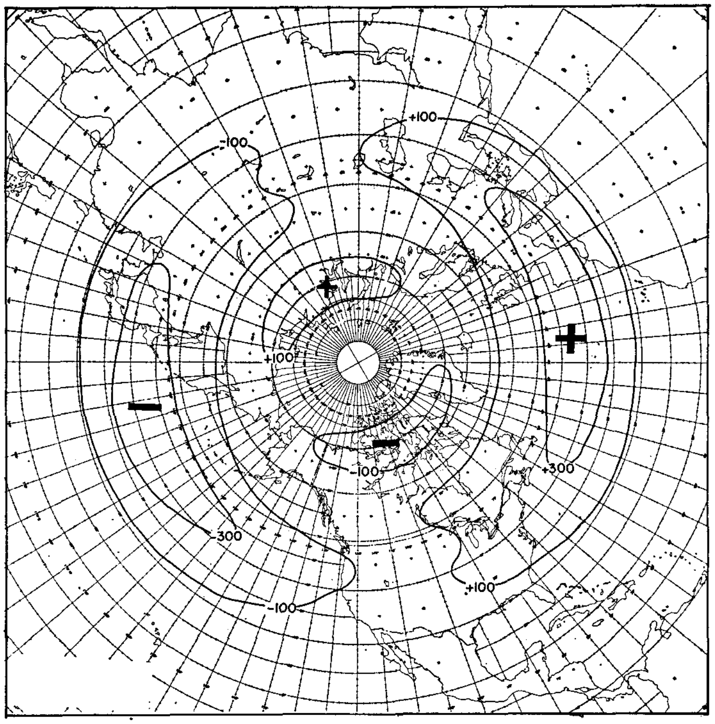


FIGURE 2.—Wave number one, in feet. Initial data, 0000 GMT, January 6, 1958.

of the error over the Pacific varies it always has a positive sign in the 48-hour forecasts. Similarly the sign of the error on the west coast of northern Europe has always been negative.

Some other persistent errors of smaller scale and slightly less persistence should also be noted in figure 1. These patterns are continental in scale and have positive centers off the east coasts of North America and Asia and negative centers inland from the west coasts of North America and Europe.

The possibility of mathematical systematic error was rejected after two numerical experiments. The data were reversed on the grid in the first experiment. This changes the direction of scan during forecast computation. In the other test the relaxation limit was lowered from ½ to ¼ foot. In neither experiment did the long-wave error change appreciably.

These observations led to the hypothesis that the errors were due to improper treatment in the prediction model of waves of very long wavelength and low wave number. To test this hypothesis a numerical method for computing the large-scale component was devised and applied to many 500-mb. initial charts and forecasts from the winter of 1957–58. This paper gives evidence in support of the following observations and conclusions derived from this investigation:

a. There is a large amount of energy present in low wave numbers in the atmosphere and these waves are quasi-stationary.

b. In current numerical models these ultra-long waves are moved westward at very great speeds and altered in shape and intensity.

c. Numerical forecasts made with these components held stationary are consistently superior to those prepared without this modification.

## 2. METHOD OF COMPUTATION

The sinusoidal form has been frequently assumed for atmospheric wave disturbances and the Fourier representation was chosen in this investigation for ease in machine computation. The form used was

$$\Phi(\lambda)=A_0+\sum_{n=1}^{n=\infty}(A_n\cos n\lambda+B_n\sin n\lambda)$$

where $\Phi$=height of 500-mb. surface

$\lambda$=longitude in radians

$n$=wave number

Computations of $A_n$ and $B_n$ were made for wave numbers one through five for each initial chart and 48-hour forecast for January and February 1958. Due to an insufficient number of grid points, the computations for wave numbers higher than one were terminated at latitude 70° N.

## 3. DESCRIPTION OF WAVE ONE IN THE ATMOSPHERE

Of the components computed, wave number one had the greatest intensity. For these 60 winter days there were two pairs of centers on each initial chart of wave number one. Since the Fourier computation enforces symmetry in each pair, it is sufficient to describe the negative center—the positive center completing the pair, being the mirror image of the negative one.

The lower latitude centers were fixed in phase with the negative center over the western Pacific Ocean. These centers varied in intensity from 300 to 850 ft.

There was another pair of centers in higher latitudes. These centers were almost randomly distributed in phase. They varied in intensity from 150 to 1,000 ft. Figure 2 shows wave one for 0000 GMT, January 6, 1958. This was the weakest wave number one pattern observed in the two-month period.

## 4. DISTORTION OF WAVE NUMBER ONE IN THE NUMERICAL FORECASTS

The basic equation of the forecast model requires the individual conservation of vorticity. From this same consideration Rossby [1] derived his famous equation governing the motion of long waves on a zonal current. The phase-speed is given by the relation:

$$C=U-\frac{\beta L^2}{4\pi^2}$$

where $U$ is the mean zonal wind, $\beta$ is the variation of the Coriolis parameter with latitude, and $L$ is the wavelength.

From this formula it is found that the maximum value of $U$ observed in the atmosphere will give eastward displacement only for wave number five and higher. There-

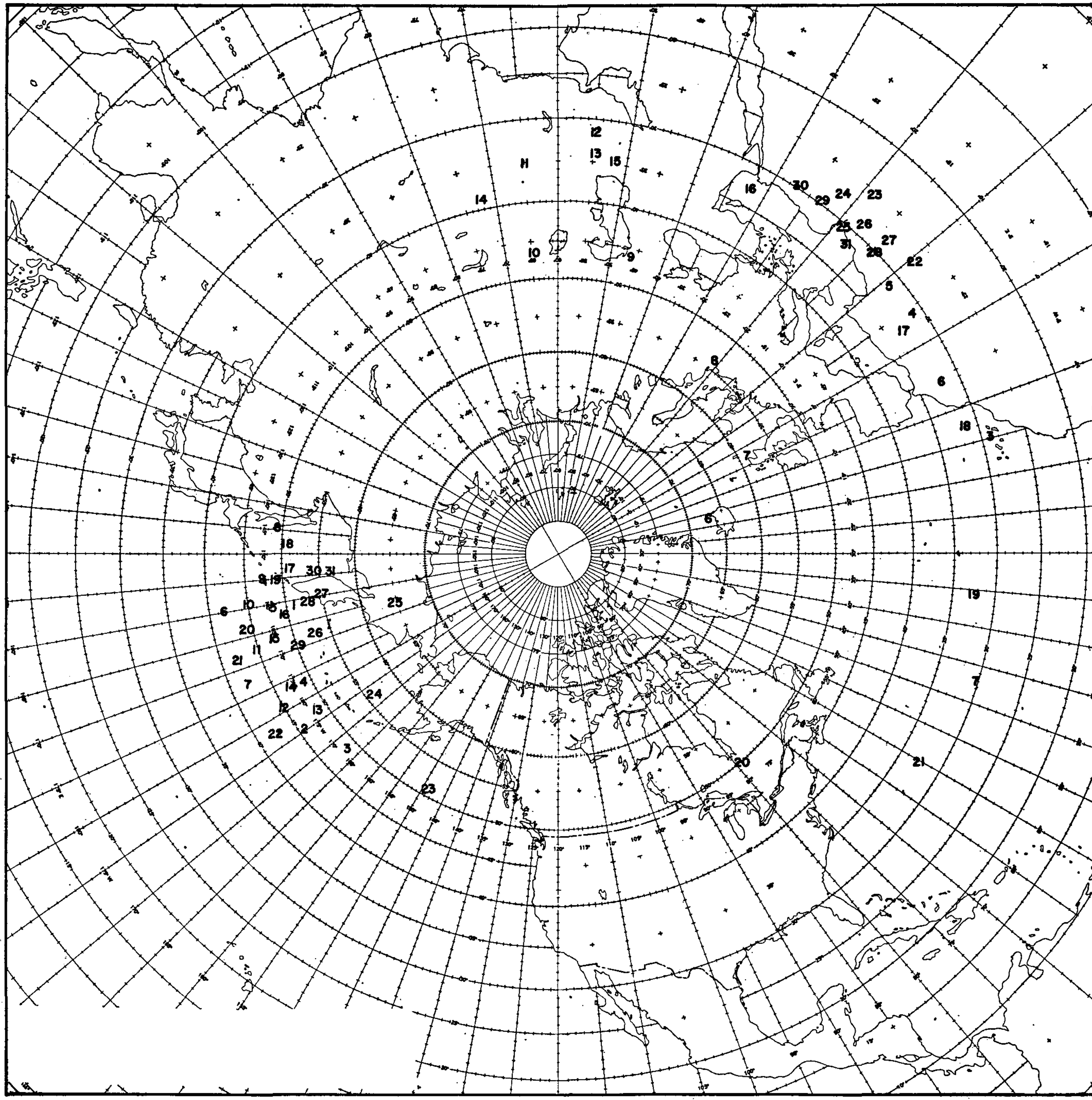

FIGURE 3.—Southern negative center, initial and 48-hour forecast positions identified by day for January 1958.

fore in this numerical model wave numbers one through four will be moved rapidly westward. In his computation of "influence functions" this effect was dismissed by Charney [2] with the statement ". . . these [very long waves] are associated with little of the total energy."

Figure 3 shows the initial and 48-hour forecast position of the southern negative center of wave number one for January 1958. The initial positions are all clustered in the western Pacific, illustrating the almost stationary nature of these waves. The forecast centers have been moved southward slightly and westward at high speed. Table 1 summarizes the treatment of both centers in the 48-hour forecasts.

TABLE 1.—*Wave one average values*

| | January | | February | |
|---|---|---|---|---|
| | D 00 | D 48 | D 00 | D 48 |
| **Southern Centers** | | | | |
| Latitude | 48 | 35 | 44 | 30 |
| Longitude | 168E | 014E | 172W | 69E |
| Intensity | 570 ft. | 280 ft. | 460 ft. | 240 ft. |
| **Northern Centers** | | | | |
| Latitude | 73 | 73 | 68 | 69 |
| Intensity | 480 ft. | 620 ft. | 440 ft. | 600 ft. |

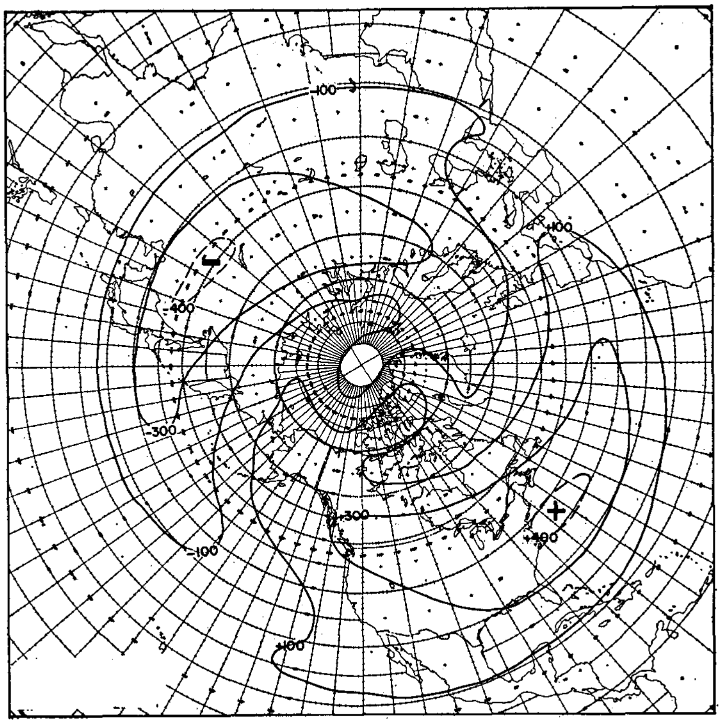


FIGURE 4.—Wave number one, in feet. 24-hour forecast from 0000 GMT, January 6, 1958.

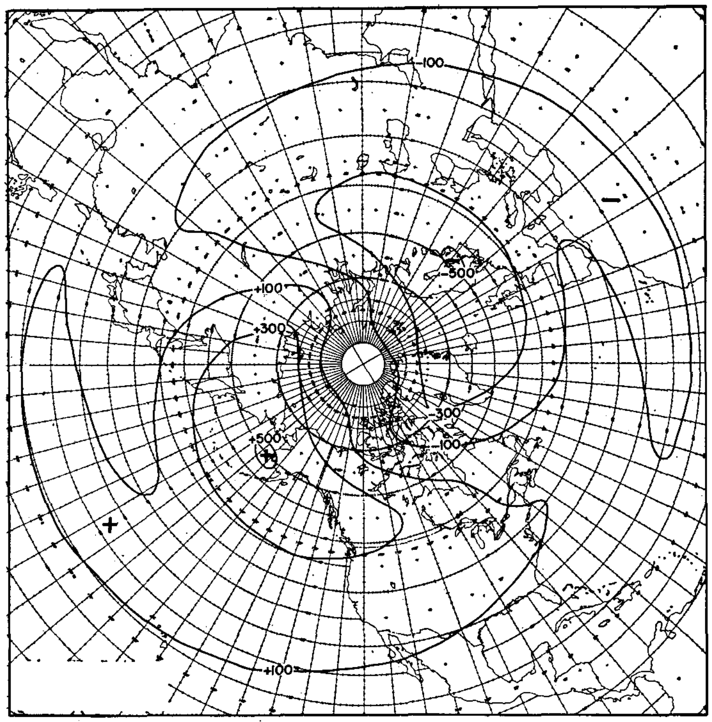


FIGURE 5.—Wave number one, in feet. 48-hour forecast from 0000 GMT, January 6, 1958.

Returning to January 6 data, figures 4 and 5 show the retrogression of wave number one at 24 hours and 48 hours in the forecast from this initial data. This January 6 initial chart is the weakest wave number one from the 60-day investigation. This false retrogression is easily seen to be a major contributor to the 48-hour error, owing to the almost exact reversal of phase.

## 5. WAVE NUMBERS 2, 3, 4, AND 5

Bristor [3] has recently developed a convenient form of machine computation of kinetic energy. This analysis was made for the individual wave components for January and February. A summary of the distribution of energy among wave numbers is shown in table 2.

Table 2 shows a surprising amount of energy rather uniformly distributed in these low wave numbers. The familiar long waves (wave number circa 5) have less important intensity.

The barotropic model occasionally held one or two of these waves stationary but spurious retrogression was the normal behavior. The slower westward motion in the forecast was somewhat balanced by the shorter wavelength so that reversal of phase was again possible in these wave numbers. Thompson [4] has shown that particular forms of zonal wind profiles are capable of holding waves of these lengths stationary.

TABLE 2.—*Average kinetic energy (units proportional to knots* $^2$*)*

| | Total perturbation KE | Wave number | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 and up |
| January | 426 | 77 | 53 | 41 | 59 | 39 | 157 |
| February | 486 | 66 | 40 | 45 | 60 | 30 | 245 |

Figure 6 is the analysis of wave number two for January 6 initial data and figure 7 is wave number three for the same day. This wave number two is somewhat more intense than average while the wave number three is about average intensity.

## 6. COMPARATIVE VERIFICATION DATA

To correct the operational forecasts for these effects, wave numbers one, two, and three were computed and added to form an approximate stationary component. The mechanics of correcting the forecast consists of operating on the stream function periodically during the forecast with the following identity.

$$\psi_c = \psi_\gamma + S_{00} - S_\gamma$$

where $\psi_c$ is corrected stream
$\psi_\gamma$ is forecast stream at time $\gamma$
$S_{00}$ is initially computed stationary components
$S_\gamma$ is stationary component at time $\gamma$

This correction was applied to a series of forecasts and was incorporated in the operational computations on April 10, 1958. To avoid excessive stabilization of the flow pattern at the high latitudes, the stabilization of wave numbers 1, 2, and 3 was terminated at latitudes 75° N., 65° N., and 55° N., respectively.

A series of forecasts was corrected for waves one, two and three. Table 3 lists the root mean square errors in

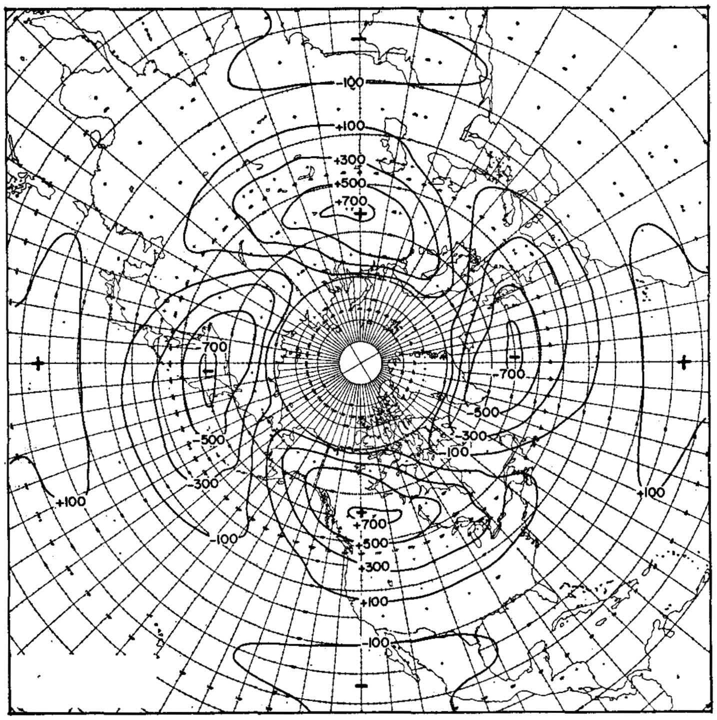

FIGURE 6.—Wave number two, in feet. Initial data, 0000 GMT, January 6, 1958.

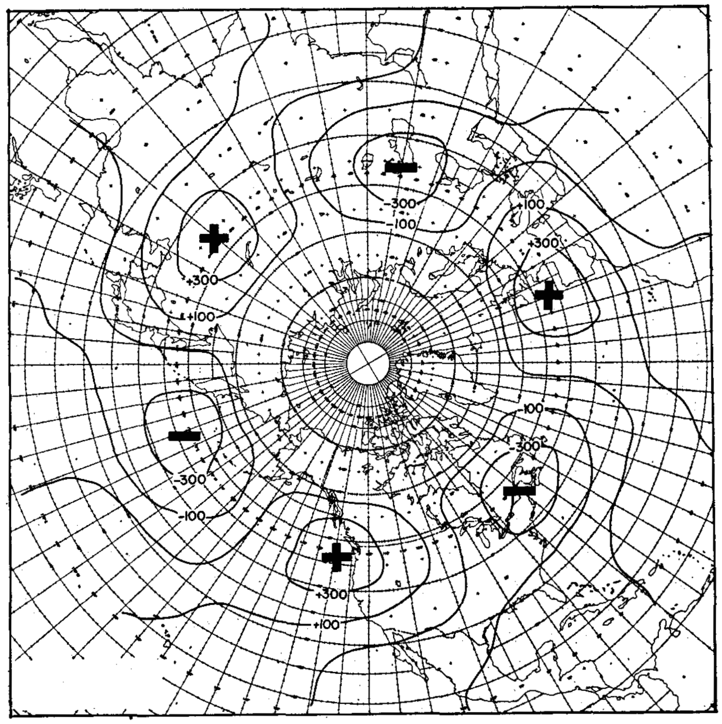

FIGURE 7.—Wave number three, in feet. Initial data, 0000 GMT, January 6, 1958.

feet for these corrected forecasts along with the original error and the error of persistence.[2] Petterssen [5] has recently suggested the comparison of root mean square errors of forecast and persistence as a valid scheme of verification.

Figure 8 shows the same type of comparison for a forecast made from 1200 GMT March 31, 1958. These curves show positive skill for the corrected forecast out to 72 hours compared with persistence.

## 7. IMPROVEMENT IN THE EMPIRICAL PROCEDURE

Figure 9 shows the error pattern for the same forecast as figure 1 after correction for the stationary component as defined in section 6. In addition to the marked reduction in error the scale of the error pattern is much more acceptable.

Although these results may appear impressive, further improvement is expected from a different method of defining the stationary component. The symmetry of the Fourier computation must introduce local errors even while effecting an over-all improvement. The discontinuity at 70° N. Lat. may produce gradient errors in high latitudes. Since the removal of the errors arising from spurious retrogression of the ultra-long waves, certain systematic errors of treatment of the zonal profile have become relatively more prominent. Improvements in the definition of the stationary component should be ac-

[2] Essentially similar results were obtained by L. Carstensen who isolated the stationary component by repeated smoothing and subtracted a tendency computed from this component at each time step in the forecast.

TABLE 3.—*Root mean square errors (feet)*

| Day | Persistence | Operational | WV1 | WV1, 2, and 3 |
|---|---|---|---|---|
| Jan 8 | 325 | 406 | 278 | 230 |
| Jan 10 | 324 | 377 | 251 | 201 |
| Jan 12 | 296 | 411 | 293 | 229 |
| Jan 14 | 309 | 414 | 296 | 252 |
| Jan 16 | 352 | 482 | 319 | 217 |
| Feb 8 | 272 | 339 | 262 | 181 |
| Feb 12 | 310 | 372 | 339 | 217 |
| Feb 16 | 303 | 479 | 320 | 227 |
| Average | 311 | 410 | 295 | 219 |

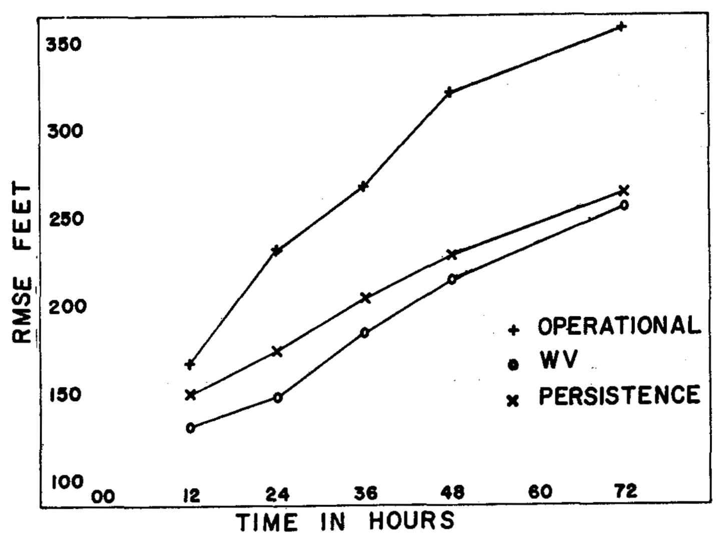


FIGURE 8.—Root mean square error curves for forecasts from 1200 GMT, March 31, 1958.

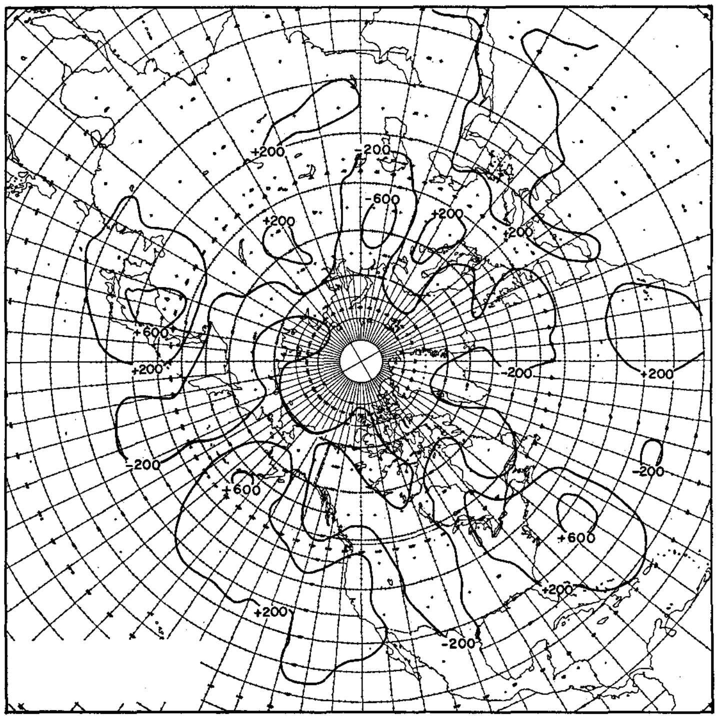


FIGURE 9.—48-hour error in feet. Operational JNWP Unit forecast corrected for waves number 1, 2, and 3 from 0000 GMT, January 16, 1958.

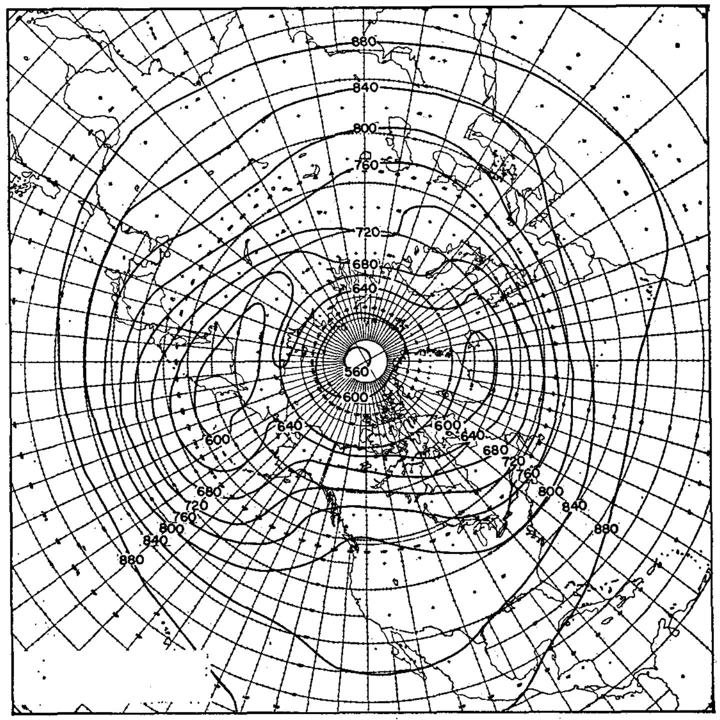


FIGURE 10.—Zonally averaged heights plus waves number 1, 2, and 3 from 0000 GMT, January 6, 1958.

companied by some method of removing systematic profile errors.

## 8. IMPROVEMENT IN THE PHYSICAL MODEL

The empirical corrections now employed should obviously be replaced by terms in the equations which describe the mechanism by which energy is transferred to and from these waves and the fields which hold them stationary in the atmosphere.

Figure 10 is the long-wave component plus the zonally averaged flow for January 6, 1958. Both this quasi-stationary component and the error locations suggest that the physical mechanism by which the large-scale quasi-stationary components are maintained must in some way reflect differences in the surface characteristics of land and sea.

A test for any formulation of the physical mechanism is a numerical prediction model which will produce forecasts superior to those obtained by the present method.

## ACKNOWLEDGMENTS

The investigation reported here would not have been possible except for the continued encouragement and guidance of Lt. Col. Philip Thompson, Head of the Development Section, JNWP Unit. The author is also indebted to LCDR W. E. Hubert, who noted the regular retrogression of the 12-hour error patterns in October 1957 and also read the completed manuscript. R. E. Walker, AG2, USN, assisted in the many computations involved.

## REFERENCES

1. C.-G. Rossby and collaborators, "Relation between Variations in the Intensity of the Zonal Circulation of the Atmosphere and the Displacements of the Semi-Permanent Centers of Action," *Journal of Marine Research*, vol. 2, No. 1, 1939, pp. 38–55.
2. J. G. Charney, "On a Physical Basis for Numerical Prediction of Large-Scale Motions in the Atmosphere," *Journal of Meteorology*, vol. 6, No. 6, December 1949, pp. 371–385.
3. C. L. Bristor, Unpublished manuscript, JNWP Unit Technical Series.
4. P. D. Thompson, "The Propagation of Permanent-Type Waves in Horizontal Flow," *Journal of Meteorology*, vol. 5, No. 4, August 1948, pp. 166–168.
5. S. Petterssen, "On Verification of Prognostic Charts," University of Chicago, *Scientific Report* No. 9, 1956, 14 pp.